Cellular Bucket Brigades
A Self-Balancing Scheme for U-Shaped Production Lines

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A **cellular bucket brigade** is a way to coordinate workers in a production line where work content is distributed on both sides of an aisle.

Consider a production line as follows. Each worker $i$ works with a speed $v_i$ in the forward direction, and works with a speed $u_i$ in the backward direction.
Hand-off

Instead of completing a job individually, workers transfer their work through ‘hand-offs’. For a two-worker system, a hand-off occurs when worker 1 moving forward meets worker 2, who is moving backward, along the aisle (point $x^t$ along the aisle). The workers exchange their work in the hand-off by crossing the aisle with a speed $w$. 
After the hand-off, worker 1 (blue) works backward with a speed $u_1$ and worker 2 (red) works forward with a speed $v_2$. 
When the workers reach the ends, they make a U-turn by crossing the aisle again.
After the U-turn, worker 1 (blue) works forward with a speed $v_1$ and worker 2 (red) works backward with a speed $u_2$. The workers meet again at point $x^{t+1}$ along the aisle.
Self-balance of cellular bucket brigades

If the hand-off location $x^t \to x^*$ as $t \to \infty$, then each worker will eventually work in a loop or a ‘cell’. We say the system self-balances.
Self-balance of cellular bucket brigades

Here is an example of a three-worker system in the balance.

\[ \begin{align*}
0 & \quad x_1^* & \quad x_2^* & \quad \frac{1}{2} \\
Start & \quad & \quad & \\
End & \quad & \quad & \\
\end{align*} \]

**Theorem:**
A cellular bucket brigade self-balances if

\[ \frac{1}{v_1} - \frac{1}{u_1} > \frac{1}{v_2} - \frac{1}{u_2} > \cdots > \frac{1}{v_n} - \frac{1}{u_n}. \]


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Consider a **special case** where each worker $i$ works with a speed $v_i$ in the forward direction, and works with a speed $\lambda v_i$ in the backward direction.

- If $\lambda > 1$ then workers should be sequenced such that $v_1 < v_2 < \ldots < v_n$. If forward line has more work, then sequence the workers from slowest to fastest in the forward direction.

- If $\lambda < 1$ then workers should be sequenced such that $v_1 > v_2 > \ldots > v_n$. If backward line has more work, then sequence the workers from slowest to fastest in the backward direction.
Given the same team of workers, a cellular bucket brigade is more productive than a serial bucket brigade if the team size $n$ and the aisle width $a$ are small.


https://ink.library.smu.edu.sg/lkcsb_research/3202/
The same idea can be applied to a situation where each worker $i$ spends times $h^+_i$ and $h^-_i$ in a hand-off with his predecessor (worker $i-1$) and successor (worker $i+1$) respectively.


https://ink.library.smu.edu.sg/lkcsb_research/4544/
The idea can also be applied to **warehouse order-picking** where each worker can pick products from both sides of an aisle.


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Applying the ideas on U-lines

U-line with discrete work stations

Dynamic U-line balancing

Assumptions:

1. Only one worker can work on a station at any time
2. Worker $i$ works on stage $j$ with velocity $v_{ij}$
3. Instantaneous walk


https://ink.library.smu.edu.sg/lkcsb_research/3512/
**Cellular bucket brigade rules on a U-line**

**Worker 1:**
- Assemble a new item on Station 1 until you meet your colleague.
- Assemble your item on Station 3 until you finish.

**Worker 2:**
- Assemble your item along the line until you meet your colleague.
References

  ▶ https://ink.library.smu.edu.sg/lkcsb_research/3202/

  ▶ https://ink.library.smu.edu.sg/lkcsb_research/3171/

  ▶ https://ink.library.smu.edu.sg/lkcsb_research/3512/

  ▶ https://ink.library.smu.edu.sg/lkcsb_research/4544/